

**2018 NDIA GROUND VEHICLE SYSTEMS ENGINEERING AND TECHNOLOGY
SYMPOSIUM
MODELING & SIMULATION, TESTING AND VALIDATION (MSTV) TECHNICAL SESSION
AUGUST 7-9, 2018 – NOVI, MICHIGAN**

**PRACTICAL FEATURES EXTRACTED FROM FULL SURFACE
TERRAIN SCANS**

Brian Liswell

Automotive Instrumentation Division
Aberdeen Test Center
Aberdeen Proving Ground, MD

ABSTRACT

Test course characterization has long relied on single-line profile measurements which provide elevation as a function of distance. These profiles are analyzed to provide various statistics and metrics. While these metrics can be useful, single-line profiles will always lead to a limited characterization. A vehicle has multiple concurrent inputs from the ground, inducing not just vertical excitations but also pitching, rolling, and twisting displacements (amongst others). Improvements in profiling equipment have enabled the ability to sample and characterize the entire surface. This paper identifies two characterization methods which take advantage of a full surface scan. The first uses orthogonal transverse modes which could either be extracted with Singular Value Decomposition (SVD) or be predefined polynomials. The second extracts a concurrent profile under each wheel for a given vehicle axle spacing and track width. Orthogonal basis vectors are then projected onto the concurrent profiles to extract the heave, pitch, roll, and twist inputs from the ground into the vehicle.

INTRODUCTION

Single-line elevation profile characterization of vehicle test courses has a long history, starting with rod and level staff survey data and progressing toward geometric and inertial based profilers. These profiles are used by different groups such as field testers, vehicle modelers, test course maintainers, and test course developers for different reasons. Procedures exist to describe the equipment and processes by which single-line profiles are generated and analyzed [1]. However, the metrics typically generated from single-line profiling at best describe the plausible input to a single vehicle wheel. The goal of this paper is to move towards establishing metrics which consider the full width of the test course or account for concurrent inputs at all wheels of the vehicle.

For this paper a single-line profile is defined as a profile intended to describe the elevation of the terrain along that line, regardless of source of the profile data. The need to move beyond single-line profile analysis is especially apparent with off-road environments. A vehicle's responsiveness to heaving, rolling, pitching, twisting and cresting inputs determines its mobility and ride quality performance. Characterizing full-width and concurrent wheel inputs extends characterization and analysis, just as half-car and full-car computer models provide advantages over a quarter-car model. This paper discusses two approaches for characterizing

terrain beyond single-line profiling. The first focuses on transverse profiles projected onto orthogonal basis vectors. The second approach focuses on concurrent vertical inputs from the ground into the vehicle wheels.

Laser Scanning Test Courses

In 2012 Aberdeen Test Center began to regularly measure and characterize its courses with a laser scanner profiler system mounted on the back of a High Mobility Multi-purpose Wheeled Vehicle (HMMWV). The system produces a data point cloud covering an area approximately 12 ft wide that extends the entire length of the course. The laser scanner simultaneously samples approximately 4,000 points on the 12-ft wide scan line, with a typical scan rate of 1,000 Hz. The point cloud density is dependent on the traveling speed, which typically ranges from 5 to 25 mph (depending on the test course). A differential GPS-based Inertial Navigation System (INS) is mounted on the same platform as the laser scanner and its outputs are recorded with the same time basis. The INS is used to measure the dynamic motions of the platform to rectify the relative distance measurements from the laser scanner into absolute ground-height measurements. Traditional single-line profiles can be extracted from the point cloud in post processing and analyzed for Wave Number Spectra (WNS), root mean square (RMS), higher order statistics, grade, and International Roughness Index (IRI) values. However, as noted earlier, these metrics do not fully describe terrain inputs that affect vehicle dynamics and ride.

TRANSVERSE PROFILES AND DERIVED CHARACTERISTICS

A more complete set of metrics for describing test courses can be produced by including transverse profiling. Two methods of transverse profiling are discussed below, including data projection onto orthogonal polynomial basis vectors and Principle Component Analysis (PCA). In both cases the larger point cloud is reduced to a simpler representation of the transverse profiles along the course length. Both methods require a uniform mapping of the laser profiler point cloud data, which in its raw form is non-uniform due to curves in test courses and time based data sampling on a platform with varying speed. The point cloud data reduction method known as Curved Regular Grid (CRG) [2] provides the uniform mapping and is described first.

Curved Regular Grid Data

A CRG maps the [X (easting), Y(northing), and Z (elevation)] coordinates of a point cloud into (U,V,Z) coordinates, where U is the distance along a curving path, V is the distance perpendicular to that path, and Z is the same height value. An explanatory graphic is provided in Figure 1. The data is then interpolated onto a grid covered by the remapped point cloud. This grid has a width that is equal to the scan width (12 ft) and a length defined by the test course length. The new rectangular area avoids dealing with the irregular empty space apparent in XYZ space. The organized nature of the gridded data permits the use of established analytical techniques such as matrix multiplication, Fast Fourier Transforms (FFT), low-pass and high-pass filtering, and cross path analysis tools (covariance (cov) and Cross Spectral Density (CSD)). For this work, a CRG is created from a decimated data point cloud to form a $L \times T$ grid, where T is the number of gridlines across the full scan width (typically 121 gridlines) and L depends on the length of the scan. The curved path is derived from the profiler INS location, though any arbitrary path could be driven through the point cloud. For this paper the CRG are detrended along the course length with a high pass Butterworth filter. Detrending is not a necessary step for any analysis described in this paper, but it is helpful for results comparison and plotting purposes.

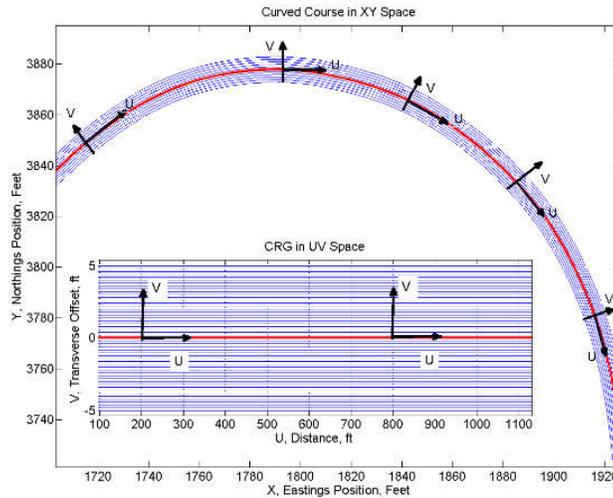


Figure 1. Graphic explaining CRG conversion from XY space to UV space

Transverse Component Analysis of CRG

Once the CRG is established the transverse profiles across the width of the course can be analyzed just as easily as the longitudinal profiles along the course length. Ferris and others [2-3] have explored projecting the transverse profiles onto orthogonal basis vectors to create independent resultant vectors. A set of basis vectors can be considered orthogonal when all the vectors are orthogonal to all other vectors in the set. Two vectors are orthogonal when their dot product equals zero. Legendre and Chebyshev polynomials are often recommended as basis vectors. In this approach the polynomials are projected against the transverse CRG grid lines (V). The first five Chebyshev polynomials are presented in Figure 2.

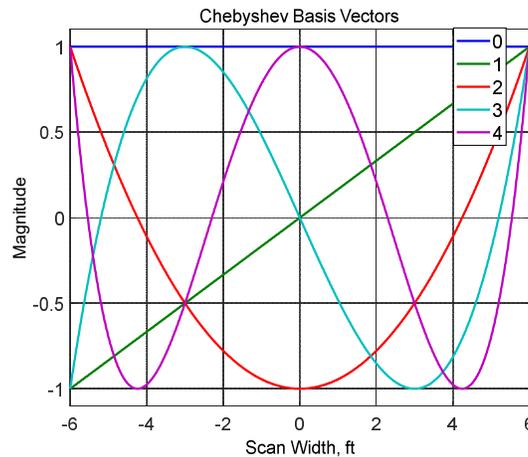


Figure 2. The first five Chebyshev polynomials

The equation for a scalar dot product projection is given in Eq. 1,

$$R_r = P \cdot Z_r \quad (1)$$

where Z_r is the original data from the CRG at row location r along length U_r , P is the unit vector onto which it is projected, and R_r is the resultant projection. This demonstrates that the projection of a CRG onto a set of basis vectors can be done with basic matrix multiplication, as shown in Eq. 2.

$$W = ZB \quad (2)$$

Each column of the resultant matrix, W , contains the changing projection of a basis vector onto a transverse line across the course width, with the basis vectors contained in the columns of B . W has the same number of columns as B and the same number of rows as the CRG, Z .

The Legendre and Chebyshev polynomials curves are useful because they can describe aspects of a road that are commonly of interest. For example, the 0th polynomial generally describes the elevation across the width, the 1st polynomial describes banking, and the 2nd describes crowning, while the 3rd and 4th describe rutting that may form along the length of the road. (An infinite number of polynomials could be calculated, with higher order polynomials consisting of more peaks-valleys and zero crossings.) To make use of this parlance, a relatively flat test course that contains a banked turn halfway along its length would have a resultant array, W , with the second column containing near zero values during the flat portions and non-zero values during the banked turn. Similarly an improved gravel road with a built-in crown would have negative values on the third column of W to indicate the crown is raised in the middle.

A potential shortcoming of the use of Legendre or Chebyshev polynomials is that the shape of higher order polynomials may not provide good fits for existing course features. For example, the “rutting” positions of the 3rd and 4th polynomials are located at 50% and 70% of the distance from the centerline. A test course may have rutting, but if the rutting does not occur at those distances from the centerline the 3rd and 4th polynomials will not represent the actual rutting very well.

Principal Component Analysis of CRG

Principal Component Analysis is an alternative means for producing transverse profiling metrics. Different methods exist, but a simple and effective technique is to perform a Singular Value Decomposition (SVD) of the covariance matrix of CRG, Z , as described in Eq. 3 and Eq. 4,

$$C_{ij} = E[(Z_i - \mu_i)(Z_j - \mu_j)] \quad (3)$$

$$C = BSA^* \quad (4)$$

where C_{ij} is the covariance between the i^{th} and j^{th} column of Z , $E[]$ is the expectation operator, and μ_i is the mean of the i^{th} column. The SVD represented in Eq. 4 describes how C (a square matrix) is decomposed into matrices B , S , and A , with $*$ denoting the complex conjugate of A . The math related to Eq. 3 and Eq. 4 do not need to be described here, except to say they can easily be performed in Matlab[®] with the commands $C=cov(Z)$ and $[B,S,A] = svd(C)$. The important thing to note is that B represents a set of unique orthonormal basis vectors while the diagonal values of matrix S , the singular values, represent the relative weightings of those vectors. Specifically, S_{ii} represents the contribution of vector B_i to the variance of the test course. Interestingly the sum of all the values in matrix C_{ij} equals the sum of all the singular values in vector S_{ii} .

The matrix B can be used to analyze changes in transverse content along the length, just as the Chebyshev polynomial vectors were used in the previous section. To illustrate this, the PCA of a test course was performed. A rendering of a test course section is presented in Figure 3, with its first four principal

components in Figure 4. The singular values for the first 20 components is presented in Figure 5. The projections of the first and third PCA components along the length of the course are presented in Figure 6. The second component essentially represents the elevation changing uniformly across the course width, which is greatly influenced by long wavelength content. There is some similarity between these components and the Chebyshev polynomials. Unlike polynomials, basis vectors from PCA are specific to the terrain being characterized. Features particular to that terrain can be captured by PCA and poorly represented by polynomials.

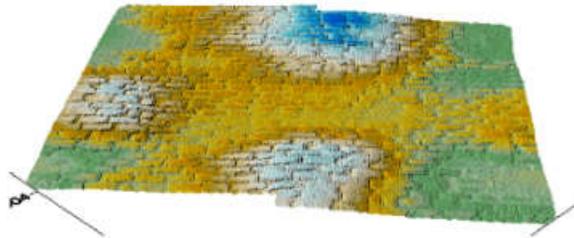


Figure 3. A rendering of a test course section and the first several principal components

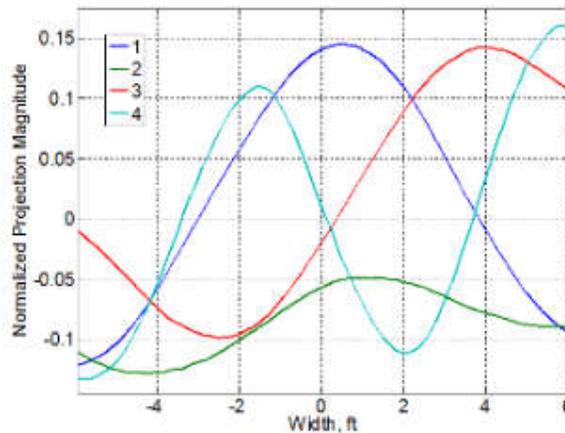


Figure 4. The first four principal components from the PCA of a test course

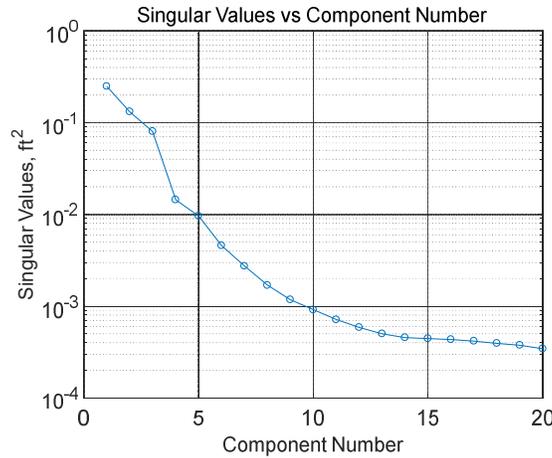


Figure 5. The singular values from the first 20 components

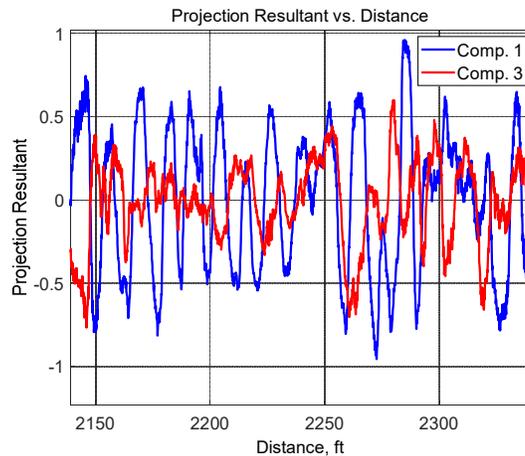


Figure 6. Projections of two components along course length

IDENTIFYING MULTIAXIAL INPUTS WITH WHEEL-BASED BASIS VECTORS

Even though the transverse profile metrics improve upon traditional single-line metrics, they do not describe how the terrain and vehicle interact. The remainder of the paper introduces a new method to generate metrics that characterize terrain-induced mode shapes associated with a test vehicles wheel base and track width. The method proposed provides insight on how test course and vehicle properties interact to excite fundamental motions of a test vehicle.

This approach starts with an arbitrary path driven through the point cloud following a reference point on a vehicle. For convenience the laser profiler INS path is often assigned as the arbitrary path. A candidate test vehicle is then selected with known wheel base and track width dimensions. The center of a simple geometric model of the candidate vehicle follows the path, where the ground height under each wheel is determined from the point cloud data. Each ground height is sampled based on the distance traveled along the path of the candidate vehicle reference point, leading to a simultaneous sampling of the input to each wheel. This means the sampling distance along each individual wheel path can be irregular, such as in curving sections where the outer wheels will have a larger sample spacing than the inner wheels. But from the vehicle reference path perspective the sampling is regular. This profiling method with irregular spacing

of wheel inputs is referred to as a Curved Regular Profile (CRP), in contrast to the CRG mapping described earlier.

A CRP may be useful to a vehicle modeler who is using advanced modeling software. But terrain analysis is usually done without access to vehicle modeling software. Simple models do exist that take terrain profiles as inputs. The IRI, a quarter car mathematical model, has been extended to half car (HRI) [4-6] and full car (FRI) [4-6] representations to take into account multiple profiles. These all make assumptions about the vehicle speed and characteristics for mass, weight, stiffness's and dampening, and distances between wheels. These models characterize the terrain by characterizing the expected response from a particular vehicle type. In contrast, this paper details how to describe inputs to a vehicle when all that is known about the vehicle is the distances between the wheels. Specifically the goal is to characterize the CRP inputs that could induce heave, pitch, roll, and twist responses in a two axle vehicle, as well as crest and double twist responses in a three axle vehicle. The process is described in such a way that the inputs for any number of axles can be identified.

Wheel-Based Basis Vectors

Basis vectors are once again used to transform a data set into an alternative representation. For this method, the data set is the wheel position elevation data from the CRP and the alternative representations are Heave, Roll, Pitch, Twist, Crest and other content. For a proper transformation that yields independent outputs, the basis vectors must be orthogonal to each other. The ordering of the wheel locations in the vector must be defined ahead of time, and the sign convention must remain consistent when verifying orthogonality. The basis vectors can be rescaled to convert the output into preferable units.

The Heave and Roll basis vectors are identified through simple inspection. Heave is defined as all elevations traveling up simultaneously. For a three axle vehicle this is represented by the non-normalized vector of [1, 1, 1, 1, 1, 1] to represent the [L1, R1, L2, R2, L3, R3] ground elevations, where L and R represents left and right, respectively, while 1,2, and 3 represent each axle counting from the vehicle front to back. Similarly, the Roll vector is identified as rolling about the center line of the vehicle, with left up and right down, or being represented as $[W_1, -W_1, W_2, -W_2, W_3, -W_3]$. W_n is defined as half the track width for the n^{th} axle.

The Pitch and Twist vectors are not identified so easily. In general, positive Pitch is defined as the vehicle front-down and rear-up, following "SAE-Z up" convention [7]. A pure Pitch action is an angled straight line running from front to rear through each axle. Twist is described as the vehicle left side having positive Pitch and the right side having negative Pitch. But what is not clear is the proper location of the Pitch axis, which is also being defined as the Twist axis in this convention. For a two axle vehicle the Pitch axis is simply the midpoint between the front and rear axle. When a third axle (or fourth, or 5th) exists the weightings given in the basis vector depend on the axle spacing. Algebraic solutions or search algorithms are used to identify vectors which minimize the Pitch vector dot product against Heave and Roll. A Modified Gram-Schmidt (MGS) algorithm is later used to ensure all final basis vectors are orthogonal to each other. The Graham-Schmidt process is a method to orthonormalize a set of vectors [8-9], and can be easily implemented with a Matlab[®] function. The initial guesses for Pitch and Twist vectors do not need to be perfectly orthogonal to Heave and Roll because the MGS algorithm will find the proper solution.

The other basis vectors for vehicles with three or more axles are also dependent on the axle spacing for their relative weightings. The number of possible basis vectors is twice the number of axles. Imagining ten basis vectors for a five axle vehicle is overwhelming. For this reason it is useful to adopt a rule for how the basis vectors should be shaped, with some guidance coming from modal analysis of free-free beams.

Consider the axle locations on one side of the vehicle to be points on a free-free beam. Each mode shape has a point of no amplitude change called a node, with a characteristic length $B_n L$ determined by the free-free boundary conditions. The units of B_n is the inverse of the unit in lengths L and x so that the term inside the trigonometric and hyperbolic functions are dimensionless. The free-free beam approach is only a tool to generate a set of orthogonal mode shapes that generally have a useful form. The MGS algorithm will slightly alter these shapes when they are included in the initial matrix with the lower order basis vectors (Heave, Roll, Pitch and Twist). An initial matrix could be created by appending random vectors to the lower order basis vectors, but the output from the MGS algorithm will appear random rather than have identifiable features such as Crest or Double Twist. The mode shape equations for a free-free beam and the characteristic length for each mode order are provided in Eqs. 5 and 6, with corresponding values provided in Table 1. The mode shape, w_n , is a function of position x , where $x = 0$ represents one end of the beam having a total length L .

$$w_n(x) = \sin(\beta_n x) + \sinh(\beta_n x) - \frac{\sin(\beta_n L) - \sinh(\beta_n L)}{\cos(\beta_n L) \cosh(\beta_n L)} (\sin(\beta_n x) + \sinh(\beta_n x)) \tag{5}$$

$$\cos(\beta_n L) \cosh(\beta_n L) - 1 = 0 \tag{6}$$

Table 1. Basis Vector Characteristics

Min. Number of Axles	Basis Vector Name		Number of nodes	Free-Free Beam Mode order, n	$B_n L$
	Symmetric	Asymmetric			
1	Heave	Roll	0	NA	NA
2	Pitch	Twist	1	0	0
3	Crest	Double Twist	2	1	4.73
4	Double Crest	Asym. Double Crest	3	2	7.85
5	Unnamed	Unnamed	4	3	11.00

Wheel-Based Basis Vectors Example

An example set of basis vectors are developed below for a three-axle vehicle with a track width of 80 in. and an axle spacing of 210 in. from axle 1 to axle 3, and 130 in. from axle 1 to axle 2. The initial vectors are not normalized for convenience, since the MGS algorithm in the final step produces normalized outputs. The vector indices order the wheel locations as [L1, R1, L2, R2, L3, R3], with the distances in front of the vehicle mid-point being positive and rearward of the midpoint being negative.

Heave and Roll are identified through inspection and the SAE Z-up convention:

$$\text{Heave, } H_i = [1, 1, 1, 1, 1, 1]$$

$$\text{Roll, } R_i = [40, -40, 40, -40, 40, -40]$$

The initial estimates for the Pitch and Twist vectors are determined from a straight line passing through each axle and the midpoint on one side of the vehicle. The slope of this line is not relevant because it is a scaling that will be removed when the vectors are normalized.

$$\begin{aligned} \text{Initial Pitch, } P_i &= [-105, -105, 25, 25, 105, 105] \\ \text{Initial Twist, } T_i &= [-105, 105, 25, -25, 105, -105] \end{aligned}$$

The Crest and Double Twist initial estimates are identified using the free-free beam model, with the recognition that to fit the form of Eq. 5 the axle spacings are normalized by the L1-to-L3 distance and shifted so the rearmost axle is set to position 0.

$$\begin{aligned} \text{Initial Crest, } C_i &= [-2.036, -2.036, 0.986, 0.986, -2.036, -2.036] \\ \text{Initial Double Twist, } D_i &= [-2.036, 2.036, 0.986, -0.986, -2.036, 2.036] \end{aligned}$$

The vectors H_i and R_i are orthogonal, and C_i and D_i are orthogonal. But the other initial vector estimates are not necessarily orthogonal. For this reason the MGS is used with the following implementation:

$$\begin{aligned} B_i &= [H_i, R_i, P_i, T_i, C_i, D_i] \\ B &= \text{mgs}(B_i) \end{aligned}$$

where the input matrix B_i contains the initial vector estimates in column form. B is the output matrix of the MGS algorithm containing the adjusted and normalized basis vectors in the matching column arrangement. For our example the labeled output of B is presented in Table 2. As can be seen, the general characteristics of the initial estimates are retained - Heave has all wheels elevating the same amount; Roll has left rising while right descends; Pitch and Twist have straight lines through the axle positions (with the height at the midpoint no longer being 0); Crest and Double Twist have the middle axle out of phase with the front and rearmost.

Table 2. Basis Vectors for Example Wheel Base

	Heave	Roll	Pitch	Twist	Crest	Double Twist
L1	0.408	0.408	-0.535	-0.535	-0.218	-0.218
R1	0.408	-0.408	-0.535	0.535	-0.218	0.218
L2	0.408	0.408	0.079	0.079	0.572	0.572
R2	0.408	-0.408	0.079	-0.079	0.572	-0.572
L3	0.408	0.408	0.456	0.456	-0.354	-0.354
R3	0.408	-0.408	0.456	-0.456	-0.354	0.354

Applying Basis Vectors

If the individual wheel profiles of the CRP are arranged as a matrix, P , with columns ordered as they are for the indices in basis vectors in B , then the multiaxial inputs as a function of distance can be calculated through simple matrix multiplication, as in Eq. 7,

$$M = PB \tag{7}$$

where each column of M contains the unscaled response to the corresponding basis vector column in B . Some representative plots of wheel path elevation profiles are presented in Figure 7, with some corresponding basis vector projections presented in Figure 8.

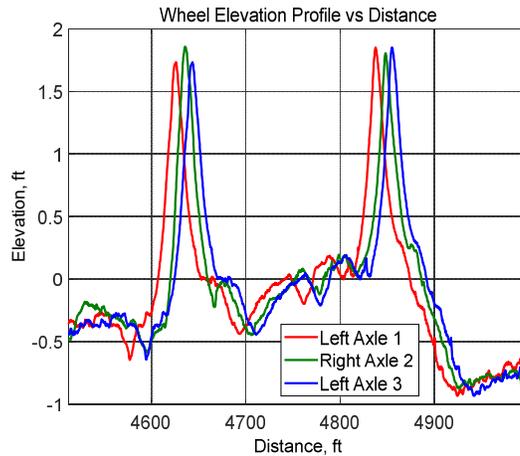


Figure 7. Example wheel path elevations

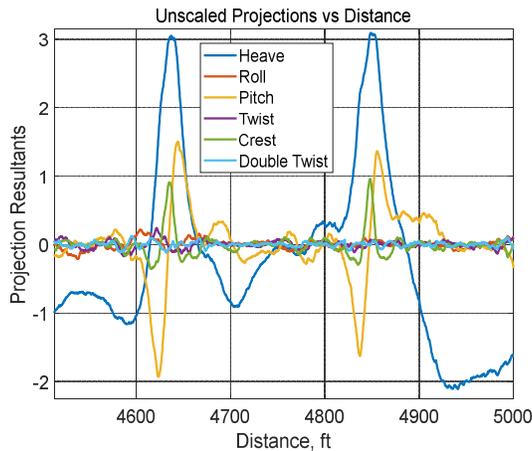


Figure 8. Projection resultants from example basis vectors applied to profiles

The content in Figure 7 shows elevation peaks approximately 200 ft apart. As can be expected the Heave resultant follows the long wavelength content as the vehicle rises and falls with the peaks. A negative Pitch results from the vehicle rising up the peak followed by a positive Pitch as the vehicle proceeds down the peak. A one-sided spike in the Crest occurs when the top of the peak is between the front and rear axles. These plots show the significance of the symmetric inputs (Heave, Pitch and Crest) and the lesser significance of the asymmetric inputs (Roll, Twist, and Double Twist). The projection resultants could be scaled to place values in meaningful units (such as feet or degrees.)

The unscaled projections from another test course are presented in Figure 9. For this course the Roll and Pitch projections have longer wavelength content while the Twist, Crest, and Double Twist content can be

seen to have higher frequency content. Decomposing the content into these projections provides insight to the stressors on the suspension and the source of the sprung mass dynamics.

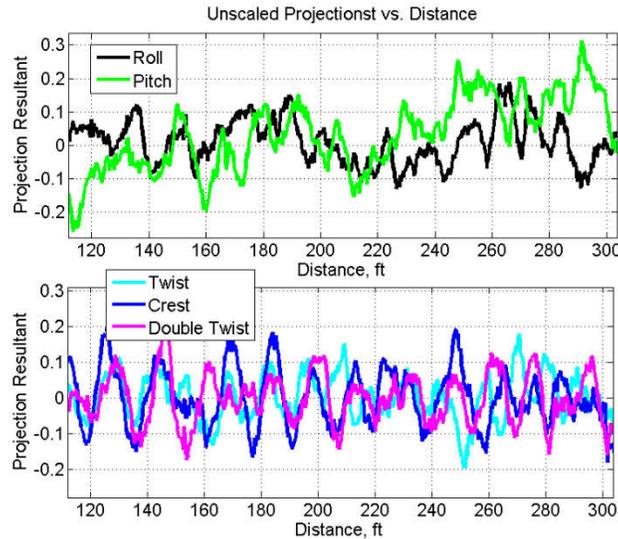


Figure 9. Projection resultants from example basis vectors applied to profiles

SUMMARY AND CONCLUSIONS

Two different approaches for expanding beyond traditional single-line profiles were presented. The first approach, which has been described in prior literature, characterizes transverse profiles from a full-width scan to assess how those transverse profiles change along the length of the course. The second approach uses the wheelbase and track width of a specific vehicle to assess the elevation inputs in terms of dynamics which could be induced in that vehicle, such as heaving, rolling, pitching, etc. Both approaches have their benefits and limitations, with each resulting in a transformed set of profiles which could be analyzed to derive beneficial metrics that speak to the content under consideration (such as a WNS or RMS relevant to rolling dynamics). The first approach characterizes the full course width, which may benefit some users, but can fail to describe the inputs that influence a specific vehicle. The second approach describes the inputs to a specific vehicle, which may benefit some users, but those inputs may not be appropriate for a vehicle with a different wheelbase (or axle count).

REFERENCES

- [1]“Test Operations Procedure (TOP) 01-1-010A Vehicle Test Course Severity (Surface Roughness)”, USATEC, APG, MD, 2017.
- [2]Chemistruck, H. M., Ferris, J.B., Gorsich, D., 2012. “Using a Galerkin Approach to Define Terrain Surfaces.” *Journal of Dynamic Systems Measurement and Control*, Volume 134, Issue 2, 021017
- [3]Ma Rui, Chemistruck, H.M., Ferris, J.B., 2012, “State-of-the-art of terrain profile characterization models,” *International Journal of Vehicle Design* (Accepted for Publication)
- [4]Sayers, Michael W. “Two quarter-car models for defining road roughness: IRI and HRI.” *Transportation Research Record* 1215 1989.

- [5]Capuruço, Renato, et al., “Full-Car Roughness Index as Summary Roughness Statistic.” Transportation Research Record: Journal of the Transportation Research Board, 1905:148–156. 2005
- [6]Alvarez, Eric J Z. “A Discrete Roughness Index for Longitudinal Road Profiles.” Masters Thesis, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, 2015
- [7]”J670: Vehicle Dynamics Terminology”, SAE International, Warrendale, PA, 2008
- [8]Moler, Cleve. Retrieved from <https://blogs.mathworks.com/cleve/2016/07/25/compare-gram-schmidt-and-householder-orthogonalization-algorithms/>, Mathworks, 2016
- [9]G. W. Stewart, “Matrix Algorithms, Volume 1: Basic Decompositions”, SIAM, 1998.