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CONFIDENCE-BASED UNCERTAINTY QUANTIFICATION AND RELIABILITY ASSESSMENT USING LIMITED NUMBERS OF INPUT AND OUTPUT TEST DATA AND VALIDATED SIMULATION MODEL

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ABSTRACT

Accurate reliability assessment requires accurate output distribution. To obtain correct output distribution, a very large number of output physical test data is required, which is prohibitively expensive. Regarding this, simulation-based methods have been developed under the assumption that: (1) accurate input distribution models obtained from large number of input test data; and (2) accurate simulation model (including surrogate model if utilized) that correctly represents physical phenomena. However, in real application, only limited numbers of input test data are available. Thus, input distribution models are uncertain. In addition, the simulation model could be biased due to assumptions and idealizations. Furthermore, only a limited number of physical output test data is available. As a result, a target output distribution to which simulation model can be validated is uncertain and the corresponding reliability is also uncertain. This paper proposes a confidence-based reliability assessment that combines uncertainty due to insufficient input/output test data and biased simulation model. To do that, a hierarchical Bayesian analysis is formulated to obtain uncertainty distribution of reliability. After that, confidence-based reliability is selected at the user-specified target confidence level. It has been numerically demonstrated that the proposed method can estimate reliability of a product that satisfies the user-specified target confidence level.

1. INTRODUCTION

The U.S. Army is continually seeking ground systems that improve performance and reliability to reduce maintenance cost significantly. It is desirable to be able to assess continual reliability of ground systems without requiring a large number of physical test data, which is very expensive. Hence, TARDEC conducts computational M&S to improve the understanding of the product performance to shape and inform program management and acquisition community for enabling rapid product development and reduced test and evaluation costs. A challenge is that reliability assessment using only computational simulation model may not be accurate.

To assess accurate reliability using the simulation model, it is assumed that (1) accurate input distribution models and (2) accurate simulation model (including surrogate model if utilized) is available. Accurate input distribution models can be constructed only when very large numbers of data for input variables are available. At the same time, the accuracy of simulation model can be verified only if a large number of physical test (output) data is available. However, in real engineering applications, it is very expensive to obtain large numbers of either input or output data. Therefore, the two conditions for accurate reliability – accurate input distribution models and accurate simulation model – may not be satisfied. Only limited numbers of input and output data are provided in engineering applications.

Thus, uncertainty arises in both input distribution models and simulation models. To consider those uncertainties, a novel reliability estimation method is developed in this paper. The paper consists of following sections. In Section 2, it is explained in detail why uncertainty due to limited number of data propagates to the uncertainty in reliability. In Section 3, the uncertainty in input distribution model is considered using Bayesian method. Section 4 describes how to capture model bias and uncertainty due to insufficient output data. In addition, both uncertainties are combined in Section 4. The developed uncertainty quantification method is verified thoroughly using an engineering example in Section 5. Finally, the findings of this study are summarized in Section 6.

2. RELIABILITY UNDER UNCERTAINTIES

Reliability of a performance measure is defined by a multidimensional integration as

$$Re(G, \boldsymbol{\zeta}, \boldsymbol{\psi}) = \int_{\mathbb{R}^N} I[G(\mathbf{x})]\, f_{\mathbf{X}}(\mathbf{x}; \boldsymbol{\zeta}, \boldsymbol{\psi})\, d\mathbf{x} \qquad (1)$$

where $\mathbf{x}$ is a realization of N-dimensional input random variable $\mathbf{X}$; $G(\mathbf{x})$ is the performance measure, which is feasible if $G(\mathbf{x}) \leq 0$; $f_{\mathbf{X}}(\mathbf{x}; \boldsymbol{\zeta}, \boldsymbol{\psi})$ is joint probability density function (PDF) of $\mathbf{X}$; $\boldsymbol{\zeta}$ is input distribution type; $\boldsymbol{\psi}$ is input distribution parameter; and $I[G(\mathbf{x})]$ is an indicator function defined as

$$I[G(\mathbf{x})] \equiv \begin{cases} 1, & \text{for } G(\mathbf{x}) \leq 0 \\ 0, & \text{otherwise.} \end{cases} \qquad (2)$$

In Eq. (1), it is shown that the inputs of reliability are performance measure $G(\mathbf{x})$, input distribution type $\boldsymbol{\zeta}$, and input distribution parameter $\boldsymbol{\psi}$. The accuracy of reliability estimation relies entirely on the accuracy of $G(\mathbf{x})$, $\boldsymbol{\zeta}$, and $\boldsymbol{\psi}$. If we do not have accurate $G(\mathbf{x})$, $\boldsymbol{\zeta}$, and $\boldsymbol{\psi}$, the reliability estimation could not be trustworthy. Moreover, if there are multiple $G(\mathbf{x})$, $\boldsymbol{\zeta}$, or $\boldsymbol{\psi}$, multiple reliability estimations are required.

If limited number of input data is used, it is not possible to obtain accurate input distribution type $\boldsymbol{\zeta}$ and input distribution parameter $\boldsymbol{\psi}$. Furthermore, the performance measure $G(\mathbf{x})$ is evaluated using a computer simulation, which may not perfectly represent real-physics. That is, the simulation model could be biased since it is an approximation of the true performance measure with assumptions and idealizations. To correct the bias, the simulation model can be validated against (output) testing. However, the output testing is usually more expensive than the input testing. Here, the insufficiency of the output data could be significant. Uncertainty arises in the simulation model $G(\mathbf{x})$ due to the bias and the limited number of output test data.

3. INPUT DISTRIBUTION MODEL UNCERTAINTY

The input distribution model, which is defined by input distribution type $\boldsymbol{\zeta}$ and input distribution parameter $\boldsymbol{\psi}$, represents variability of input random variables. To consider the uncertainty in input distribution model induced by the limited number of input data, Bayesian approach has been applied to the reliability analysis [1]. The Bayesian approach selects multiple candidate input distribution models – multiple sets of $\boldsymbol{\zeta}$ and $\boldsymbol{\psi}$ – based on the (limited number of) input data. Each candidate input distribution model produces a reliability realization. Once reliability of all candidate distributions are evaluated, distribution of reliability can be obtained.

3.1 Parameter uncertainty

The input distribution parameter $\boldsymbol{\psi}_i$ for a random variable X_i consists of input mean μ_i and input variance σ_i^2. If $\mathbf{x}_i^e$, input data for X_i, is given, the probability of σ_i^2 follows inverse chi-square distribution as [1]

$$\sigma_i^2|\mathbf{x}_i^e \sim \mathrm{Inv}-\chi^2(ND-1, s_i^2) \qquad (3)$$

where ND is the number of data and s_i^2 is the sample variance.

The input mean μ_i follows normal distribution based on $\mathbf{x}_i^e$ as [1]

$$\mu_i|\sigma_i^2, \mathbf{x}_i^e \sim \mathrm{N}(\bar{x}_i, \sigma_i^2/ND) \qquad (4)$$

where $\bar{x}_i$ is mean of data subset $\mathbf{x}_i^e$.

Realizations of $\boldsymbol{\psi}_i$ from Eqs. (3) and (4) create candidates of input distribution parameter $\boldsymbol{\psi} = \{\boldsymbol{\psi}_i | i = 1, \dots, N\}$ where N is number of random variables with (limited number of) data.

3.2 Distribution type uncertainty

Once the candidates of input distribution parameter are obtained, the probability of input distribution type $\boldsymbol{\zeta}$ can be obtained as [1]

$$P(\boldsymbol{\zeta}|\boldsymbol{\psi}, \mathbf{x}^e) = \frac{L(\mathbf{x}^e; \boldsymbol{\zeta}, \boldsymbol{\psi})}{\sum_{\mathbf{Z}} L(\mathbf{x}^e; \boldsymbol{\zeta}, \boldsymbol{\psi})} \qquad (5)$$

where $L(\mathbf{x}^e; \boldsymbol{\zeta}, \boldsymbol{\psi})$ is the likelihood function. From Eq. (5), candidates of input distribution type is obtained.

Once the candidates of input distribution model is obtained, the output distribution of each candidate can be generated using the performance measure $G(\mathbf{x})$. If $G(\mathbf{x})$ is a biased simulation model and there is only limited number of output test data, the uncertainty induced by them should be considered. This subject will be explained in the following section.

4. CONFIDENCE-BASED RELIABILITY ESTIMATION

Using candidates of input distribution models, possible biased simulation output PDFs can be obtained. Still, the possible simulation output PDFs and reliabilities from them are not accurate due to the model bias.

Characterizing the model bias is difficult task because it means obtaining true output PDF using a large number of physical output tests. Since only a limited number of output test data is available, there exists the uncertainty induced by insufficient output test data. Accordingly, a new reliability assessment method should consider the uncertainty induced by insufficient output test data as well as uncertainty of input distribution model. As a consequence, reliability cannot be uniquely determined under these uncertainties. Thus, the proposed method evaluates a confidence level; and the developed method is referred to as a confidence-based reliability assessment.

Section 4.1 explains the uncertainty due to limited number of output test data, which results in uncertain target output PDF and reliability. In Section 4.2, the uncertainty due to limited number of output test data is combined with uncertain input distribution model quantified in Section 3 and the biased simulation model. By combining all uncertainties, uncertainty of reliability can be obtained and thus the confidence level of reliability can be evaluated. Section 4.3 describes how to select the confidence-based reliability and target output PDF, which satisfy the user-specified target confidence level.

4.1 Uncertainty due to limited number of physical output test data

In the presence of the uncertainty due to insufficient output test data, a predicted output PDF becomes uncertain. As a result, there could be various possible predicted output PDFs as shown in Figure 1. Accordingly, an uncertainty also exists in the predicted reliability, which is calculated based on the predicted output PDF. However, a confidence-based target output PDF has to be selected to validate simulation output PDF against it.

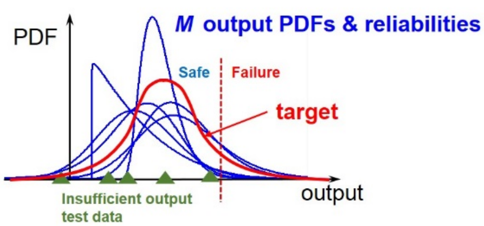


Figure 1: Uncertainty in target output PDF and reliability due to insufficient output test data

In this study, to model output PDF, adaptive kernel density estimation (AKDE) as a non-parametric method is used because the output distribution type may not belong to any standard parametric distribution. In AKDE, the bandwidth (h_0) is the only unknown parameter; hence, the uncertainty of output PDF is reflected by a posterior distribution of the bandwidth. Using the Bayesian analysis, the posterior distribution of the bandwidth $P(h_0|\mathbf{y}^e)$ given insufficient output test data, $\mathbf{y}^e$, can be obtained by the product of the likelihood function and prior distribution [2]. The information of biased simulation output PDF, which is generated using the candidates of input distribution model and biased simulation model in Section 3, is used to construct prior distribution. Then, the CDF of the reliability, $F_{Re}(Re|\mathbf{y}^e)$, can be formulated as the following

$$F_{Re}(Re|\mathbf{y}^e) = \int_0^{Re} \int_{\Omega_{h_0}} f(Re|h_0, \mathbf{y}^e)\, P(h_0|\mathbf{y}^e)\, dh_0 dRe \quad (6)$$

where $f(Re|h_0, \mathbf{y}^e) = \delta[Re - Re(h_0)]$ is the conditional PDF of reliability given bandwidth h_0, which is the Dirac delta measure. Meanwhile, the posterior distribution of bandwidth $P(h_0|\mathbf{y}^e)$ in Eq. (6) cannot be analytically obtained so that the realization of the h_0 needs to be generated using the Markov Chain Monte Carlo (MCMC) sampler in accordance with $P(h_0|\mathbf{y}^e)$. Consequently, integration in Eq. (6) is numerically evaluated using MCS as

$$\begin{aligned} F_{Re}\ (Re|\mathbf{y}^e) &= \\ &\cong \frac{1}{M}\int_0^{Re} \sum_{i=1}^{M} \left[f\left(Re|h_0^{(i)}, \mathbf{y}^e\right)\right] dRe \\ &= \frac{1}{M}\int_0^{Re} \sum_{i=1}^{M} \delta\left[Re - Re\left(h_0^{(i)}\right)\right] dRe \\ &= \frac{1}{M}\sum_{i=1}^{M} I_{[0,Re]}\left[Re\left(h_0^{(i)}\right)\right] \end{aligned} \quad (7)$$

where M is the number of MCS samples and $h_0^{(i)}$ is the i^{th} realization of h_0 by MCMC sampling. It is noted that the obtained CDF of reliability represents the uncertainty of reliability.

4.2 Combine uncertainty due to limited number of output test data with uncertainty of input distribution models

In the presence of uncertain input distribution models, many possible biased simulation output PDFs exist as explained in Section 3. Thus, single level Bayesian model that is proposed in Section 4.1 is not appropriate. Hence, a hierarchical Bayesian model is proposed to combine uncertainties induced by limited input/output test data and biased simulation model as

$$P(h_0, \boldsymbol{\zeta}, \boldsymbol{\psi} | \mathbf{y}^e, \mathbf{x}^e) \propto L(\mathbf{y}^e | h_0, \boldsymbol{\zeta}, \boldsymbol{\psi}, \mathbf{x}^e) P(h_0, \boldsymbol{\zeta}, \boldsymbol{\psi} | \mathbf{x}^e)$$

where (8)

$$P(h_0, \boldsymbol{\zeta}, \boldsymbol{\psi} | \mathbf{x}^e) \propto P(h_0 | \boldsymbol{\zeta}, \boldsymbol{\psi}, \mathbf{x}^e) P(\boldsymbol{\zeta}, \boldsymbol{\psi} | \mathbf{x}^e)$$

Here, $L(\mathbf{y}^e | h_0, \boldsymbol{\zeta}, \boldsymbol{\psi}, \mathbf{x}^e)$ is likelihood function obtained using AKDE; $P(h_0 | \boldsymbol{\zeta}, \boldsymbol{\psi}, \mathbf{x}^e)$ is the prior distribution of bandwidth given input distribution model; and $P(\boldsymbol{\zeta}, \boldsymbol{\psi} | \mathbf{x}^e)$ is hyper prior for input distribution model, which is product of Eqs. (3), (4), and (5).

After applying hierarchical Bayesian model, posterior distribution of bandwidth can be obtained in Eq. (8). It is worth noting that the posterior distribution of bandwidth is different from one obtained in Section 4.1 because it additionally considers uncertainty of input distribution models. Accordingly, many possible candidates of output PDFs can be obtained. Then, the corresponding reliability can be evaluated for each output PDF, which represents uncertain reliabilities. Many possible reliabilities can construct the CDF of reliability. To provide confidence information, complementary CDF (CCDF), which is 1−CDF, may be more useful.

4.3 Confidence-based reliability and target output distribution

The proposed method suggests to use a conservatively selected reliability value based on the CCDF of reliability. In the CCDF, higher percentile indicates more conservative estimation of reliability. This is why the percentile is referred to as the confidence level as mentioned earlier. The user can select a target confidence level CL^{target}. The reliability at the target confidence level is the confidence-based reliability as shown in Figure 2. The corresponding output PDF that produces confidence-based reliability is the confidence-based target output PDF. Once the confidence-based target output PDF is selected, the biased simulation model output PDF can be validated against it. In this paper, the validation process is left as future research.

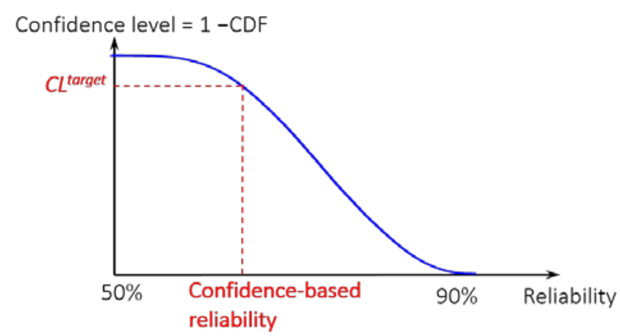


Figure 2: Confidence-based reliability and CCDF

5. NUMERICAL EXAMPLE: 11-D VEHICLE SIDE IMPACT PROBLEM

The proposed confidence-based reliability assessment is demonstrated using 11-D vehicle side impact problem [3,4]. Table 1 shows true input distributions of 11 input random variables. To represent a practical situation, it is assumed that we know true input distributions only for $X_1 \sim X_7$, which are thicknesses of steel plates. On the other hand, only limited numbers of data are available for material properties ($X_8 \sim X_9$) and crash properties ($X_{10} \sim X_{11}$). For these four input variables, ten test data is randomly drawn from true input distributions like carrying out testing. Out of ten constraints in the original problem [3,4], only three active constraints at the current design are considered as:

Constraint 1: lower rib deflection – 31.5mm ≤ 0
Constraint 2: pubic symphysis force – 3.98kN ≤ 0
Constraint 3: velocity of front door at B-pillar − 15.55mm/ms ≤ 0

Biased constraints G_i ($\mathbf{X}$) are formulated by subtracting bias $B_i(\mathbf{X})$ from true output model $G_i^{true}(\mathbf{X})$ as $G_i(\mathbf{X}) = G_i^{true}(\mathbf{X}) - B_i(\mathbf{X})$. The true outputs (*i.e.*, true physical output) are defined as

$$\begin{aligned} G_1^{true}(\mathbf{X}) &= 14.86 + (-9.9X_2 \\ &\quad -12.9X_1X_8 + 0.1107X_3X_{10}) \\ G_2^{true}(\mathbf{X}) &= 0.74 + (-0.5X_4 - 0.19X_2X_3 \\ &\quad -0.0122X_4X_{10} + 0.009325X_6X_{10} \\ &\quad +0.000191X_{11}^2) \\ G_3^{true}(\boldsymbol{X}) &= 1.5 + (-0.489X_3X_7 \\ &\quad -0.843X_5X_6 + 0.0432X_9X_{10} \\ &\quad -0.0556X_9X_{11} - 0.000786X_{11}^2) \end{aligned} \tag{9}$$

and biases for three constraints (from the simulation model) are given by

$$\begin{aligned} B_1(\boldsymbol{X}) &= 1.67X_1^{2.3} + 2.6X_2 \\ &\quad - 0.017X_8X_{10}^2 \\ B_2(\boldsymbol{X}) &= 0.16X_4^{2.4} \\ B_3(\boldsymbol{X}) &= 0.79(2X_6 - X_5) \\ &\quad - 0.00013X_{10}^2X_{11} \end{aligned} \tag{10}$$

Note that in demonstration of the proposed method, we do not know the true outputs in Eq. (9) and biases in Eq. (10). We only use the biased constraints $G_i(\mathbf{X}) = G_i^{true}(\mathbf{X}) - B_i(\mathbf{X})$ as the simulation model.

In many engineering applications, Eq. (1) cannot be analytically evaluated because output performance measure $G(\mathbf{X})$ is usually nonlinear. Thus, in this study, a sampling-based reliability analysis is used, which calculates reliability using MCS. Then, Eq. (1) can be approximated by

$$Re(G, \boldsymbol{\zeta}, \boldsymbol{\psi}) \cong \frac{1}{nMCS} \sum_{k=1}^{nMCS} I\left[G\left(\mathbf{x}^{(k)}\right)\right], \tag{11}$$

where $\mathbf{x}^{(k)}$ is k^{th} realization of $\mathbf{X}$. However, for large-scale computer-aided engineering (CAE) simulation models, direct use of MCS requires a very large number of simulations. To resolve issue, the dynamic Kriging (DKG) method [5,6], which is one of the most accurate surrogate modeling methods [7,8], is used.

Table 1: Input variable information

Description		True input distributions			Remark
		Type	Mean	STD	
X_1	B-pillar inner	Normal	0.5	0.015	True input distributions are known
X_2	B-pillar reinforce	Normal	1.3	0.039	True input distributions are known
X_3	Floor side inner	Normal	0.5	0.015	True input distributions are known
X_4	Cross member	Normal	1.3	0.039	True input distributions are known
X_5	Door beam	Normal	1.1	0.033	True input distributions are known
X_6	Door belt line	Normal	1.5	0.045	True input distributions are known
X_7	Roof rail	Normal	0.5	0.015	True input distributions are known
X_8	Mat. B-pillar inner	Log Normal	0.345	0.0242	Only limited number of data are available
X_9	Mat. Floor side inner	Log Normal	0.192	0.0134	Only limited number of data are available
X_{10}	Barrier height	Normal	0	10	Only limited number of data are available
X_{11}	Barrier hitting	Normal	0	10	Only limited number of data are available

To describe the error level of biased simulation model, the output mean of biased simulation model have been compared with those obtained using true model in Table 2. It can be seen that the bias is around 10% of true model, which indicates that the simulation model is reasonable.

Table 2: Accuracy of biased simulation model

	Output mean		
	G_1	G_7	G_9
Biased simulation model (a)	28.70	3.64	13.82
True model (b)	30.83	3.94	15.32
Error $\left(\frac{b-a}{b} \times 100\right)$	**10.16%**	**7.63%**	**9.80%**

5.1 Confidence level and confidence-based reliability

The proposed confidence-based reliability has been applied to the 11-D side impact problem. Figure 3 depicts the ten input data for each of four variables, which are randomly drawn from true input distributions. Figure 4 describes how five output test data are distributed, which are randomly drawn from true output distributions.

For simulation, the DKG surrogate models have been generated for the biased constraints $G_i(\mathbf{X})$, which are used for analysis at the design of experiment (DOE) points. The DKG models rightly cover the candidate input distributions (local window) instead of the entire design domain (global window) to reduce number of required DOE points for accurate results. The Transformation/Gibbs sampling method (TGS) is used to provide uniform initial 200 DOE samples in the local window. Then, 10 additional DOE points are sequentially added to reduce the variance of the Kriging results in between DOE sample points until acceptable accuracy has been achieved. In total, responses of 410 DOEs and 1.92 hours with Intel i7-2600 CPU and 16GB of RAM have been used for creation of the DKG models.

The CCDF of reliability is obtained as shown in Figure 5. At the target confidence level of 90%, the confidence-based reliability is evaluated as 62.78% (for G_1), 62.92% (for G_2) and 58.48% (for G_3). In addition, for the purpose of comparison, two other methods have been carried out: the simulation-based method and the method that best fits output test results. The simulation-based method uses biased simulation model and best-fit input distribution models that are approximated by maximum likelihood estimation using the ten input test data for each variable. The output best-fit method utilizes five output test data to estimate the reliability by directly applying AKDE to the data. Hence, the biased simulation model is not used for

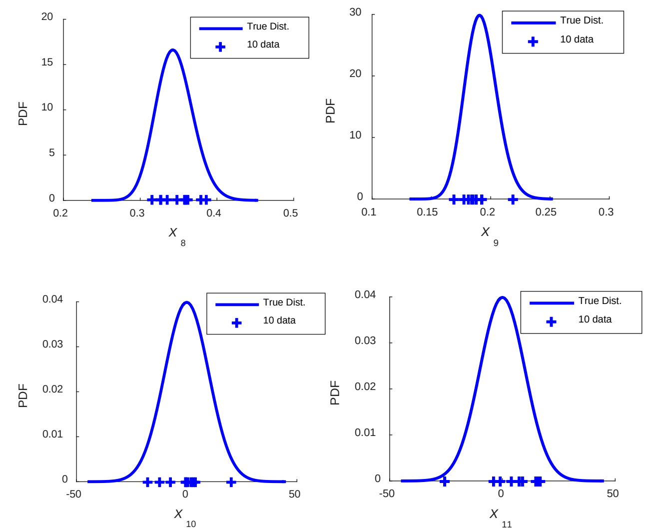
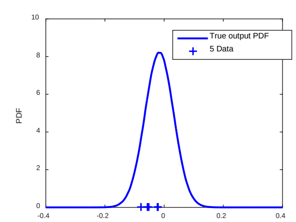
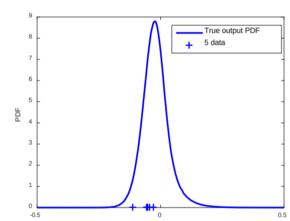


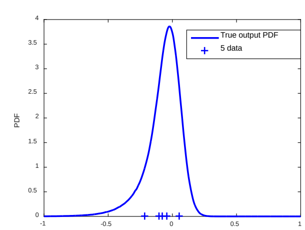
Figure 3: Ten input data drawn from true input distributions



(a) G_1



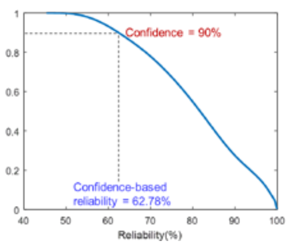
(b) G_2



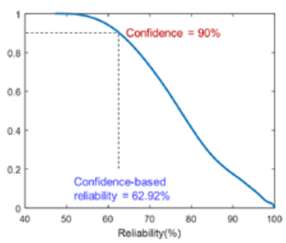
(c) G_3

Figure 4: Five output data drawn from true output distribution

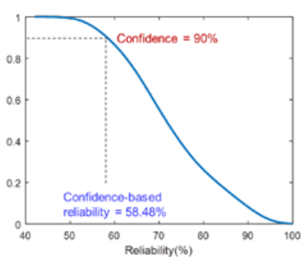
the output best-fit method. Table 3 lists confidence-based reliability, output best-fit reliability and simulation-based reliability. It can be seen that both the simulation-based reliability and output best-fit reliability overestimate the true reliability. On the other hand, confidence-based reliability is conservative compared to the true reliability. In particular, both simulation-based reliability and output best-fit reliability for G_1 and G_2 are very close to 100% which will mislead to wrong decision on product design. However, the developed method can prevent the wrong decision by providing safe estimation of reliability (*i.e.*, conservative reliability).



(a) G_1



(b) G_2



(c) G_3

Figure 5: Confidence level and confidence-based reliability

Table 3: Summary of reliability estimation

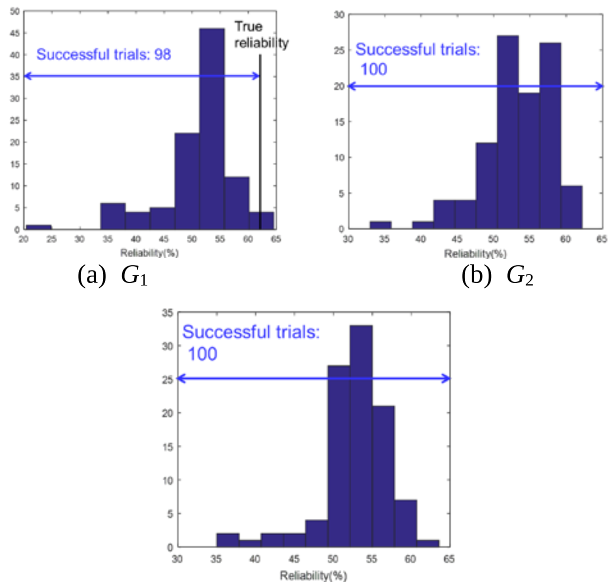
Reliability	Constraint		
	G_1	G_2	G_3
Confidence-based (using DKG)	62.78%	62.92%	58.48%
Output Best-fit	92.53%	100.00%	80.16%
Simulation-based (using DKG)	98.26%	99.82%	99.89%
True	**63.23%**	**69.08%**	**67.36%**

To check accuracy of reliabilities obtained using the DKG models, they are recalculated using the analytical functions of $G_i(\mathrm{X})$. The results are 98.26%, 99.82% and 99.90%, which are extremely close to the values in Table 3 (98.26%, 99.82% and 99.89%). Hence, we can see that the DKG surrogate models are very accurate.

5.2. Practical Demonstration of Confidence Level

In confidence-based reliability method, we have to ask a question whether the estimated reliability truly satisfies target confidence level. It is not easy to theoretically prove that the confidence-based reliability is conservative. In this section, the confidence level is numerically demonstrated by repeating 100 times of confidence-based reliability assessment with different sets of input/output test data. 100 sets of ten input test data and five output test data are randomly drawn from true input and physical output distributions, respectively. These 100 repeated tests have been carried out on the HPC system—Excalibur (60 nodes in parallel; each node has 32 cores and 128 GB memory)—at the U.S. Army Research Laboratory. One confidence-based reliability assessment takes approximately 3 hours using 540 cores and 128 GB memory. For these 100 trials, a comparison study between two methods, the proposed confidence-based method and output best-fit method, has been carried out. Figures 6 and 7 illustrate the histograms of confidence-based reliability and output best-fit reliability for 100 trials, respectively. Figures 6 and 7 show how many trials conservatively estimate (less than) true reliability for both methods. It can be seen that the confidence-based method satisfies target confidence level of 90% − 98% (for G_1), 100% (for G_2) and 100% (for G_3), whereas output best-fit method does not provide enough confidence − 51% (for G_1), 42% (for G_2) and 44% (for G_3). In addition, it can be noticed that the right-tail of histogram for output best-fit method is heavier than one for the confidence-based method.

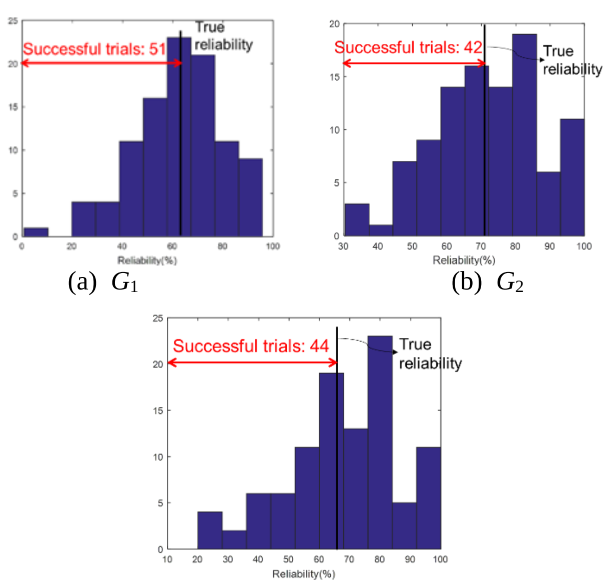
This implies that the proposed method can prevent gross overestimation of reliability.



(a) G_1 (b) G_2

(c) G_3

Figure 6: Histogram of confidence-based reliability for 100 trials



(a) G_1 (b) G_2

(c) G_3

Figure 7: Histogram of output best-fit reliability for 100 trials

6. CONCLUSIONS

In this study, we have developed a novel reliability assessment methodology which considers uncertainties due to limited number input/output test data and biased simulation model, which occur in many practical engineering applications. To combine all uncertainties, hierarchical Bayesian analysis is carried out to obtain the uncertainty distribution (*i.e.*, CCDF) of reliability that can provide confidence level of reliability. Thus, the proposed confidence-based reliability method provides the reliability estimation at the target confidence level that engineers set. It is numerically demonstrated that the proposed method satisfies target confidence level that true reliability is larger than the estimated reliability. The proposed method can be applied to any engineering problem where practical experimental situation necessitates reliability assessment with confidence.

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ACRONYMS

AKDE	Adaptive kernel density estimation
CAE	Computer-aided engineering
CCDF	Cumulative CDF
CDF	Cumulative distribution function
DKG	Dynamic Kriging
DOE	Design of experiment
MCMC	Markov Chain Monte Carlo
MCS	Monte Carlo simulation
PDF	Probability density function

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